63. $\int \frac{1}{x} = \int \frac{1}{1+(x-1)} = \int 1-(x-1)+(x-1)^2-(x-1)^3\cdots$

$\frac{a}{1-r}$ $a=1$ $r=-(x-1)$

$\ln x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} \cdots + C$

let $x=1$

$\ln 1 = 1 + C$ $C=-1$

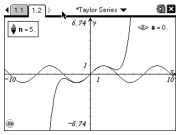
$\ln x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} \cdots$

$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} \cdots$

Feb 2-10:00 AM

9.2a Taylor Series

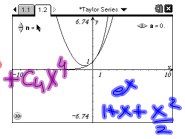
How can we find a polynomial that looks like another function?



Jan 28-10:59 AM

Ex. Find a parabola that fits y=e^x for x near 0

(tangent parabola at $x=0$)



$e^x = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4$

$e^0 = c_0$ $c_0 = 1$

f': $e^x = c_1 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3$

$e^0 = c_1$ $c_1 = 1$

f'' $e^x = 2c_2 + 3\cdot 2\, c_3 x + 4\cdot 3\, c_4 x^2$

$e^0 = 2c_2$ $c_2 = \frac{1}{2}$

f''' $e^x = 3\cdot 2\cdot c_3 + 4\cdot 3\cdot 2\, c_4 x$ $c_3 = \frac{1}{3\cdot 2\cdot 1}$

$e^0 = 3\cdot 2\cdot c_3$

f^4 $e^x = 4\cdot 3\cdot 2\, c_4 + \cdots$ $c_4 = \frac{1}{4\cdot 3\cdot 2\cdot 1} = \frac{1}{4!}$

$x=0$ $e^0 = 4\cdot 3\cdot 2\, c_4$

$c_5 = \frac{1}{5!}$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} \cdots \frac{x^n}{n!} \cdots$

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$5! = 5\cdot 4\cdot 3\cdot 2\cdot 1$

$0! = 1$

Jan 28-11:04 AM

Find an nth degree polynomial that fits y=e^x for x near 0

Jan 28-11:02 AM

Construct a 4th degree polynomial that matches y=ln(1+x) at x=0

$f(x)=\ln(1+x)$ $f(0)=0$

$f'(x)=\frac{1}{1+x}$ $f'(0)=1$

$f''(x)=\frac{-1}{(1+x)^2}$ $f''(0)=-1$

$f'''(x)=\frac{2}{(1+x)^3}$ $f'''(0)=2$

$f^4(x)=\frac{-3\cdot2}{(1+x)^4}$ $f^4(0)=-3!$

$\ln(1+x)=x-\frac{x^2}{2}+\frac{2x^3}{3\cdot2}-\frac{3\cdot2\,x^4}{4\cdot3\cdot2}$

$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}-\frac{x^6}{6}\ldots$

memorize

e^x $\sin x$

$\ln(1+x)$ $\cos x$

Jan 28-11:35 AM

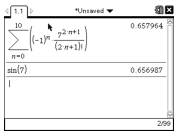
Maclaurin Series for f(x)

$f(x)=c_0+c_1x+c_2x^2+c_3x^3\ldots+c_nx^n$

$f(x)=f(0)+f'(0)x+\frac{f''(0)}{2}x^2+\frac{f'''(0)}{3!}x^3\ldots\frac{f^n(0)}{n!}x^n$

Jan 28-11:03 AM

Find the Maclaurin series for f(x)=sin(x). How many terms are required to approximate sin(7) accurate to the third decimal place?



Jan 28-11:37 AM

$\sin(x^2)\overset{?}{=}x^2-\frac{(x^2)^3}{3!}+\frac{(x^2)^5}{5!}$

$\sin(x)=x-\frac{x^3}{3!}+\frac{x^5}{5!}\ldots$

Feb 2-10:55 AM